Brief introduction to the Darrieus wind turbines

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The Darrieus wind turbines

These vertical axis wind turbines are quite sophisticated machines, so here we will only make a brief presentation of them. Their rated tip-speed ratio is \( \lambda_d = 5 \ldots 7 \).

The Darrieus rotor is built with blades with a high-performance symmetric profile, such as the NACA 0012 profile. The shape and the aero dynamical characteristics of this profile are shown in Figure 2.

They generally have between two and three blades, which can be vertical, inclined or "folded" in a semicircle or a parabola (Fig. 1).

These rotors are usually only used for wind turbines connected to the electrical grid, since they can not start by themselves.

For our readers who wish to know a little more about the aerodynamics of the Darrieus rotor, we have added two figures: Fig. 3 shows the aerodynamic forces acting on a blade element at a random position (rotation angle \( \varphi \)), while Fig. 4 shows the behavior of the three vectors of the velocities \( v' \), \( u' \) and \( c \) who attacks the blades during a complete rotation around the vertical axis, where

\[
\begin{align*}
v' &= \text{wind speed passing trough the rotor} \\
u' &= \text{relative velocity of the air on the blade element (tangential velocity of the blade element, perpendicular to the rotor radius)}
\end{align*}
\]

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\[ c = \text{absolute velocity of the blade element (resultant of the vectors } v' \text{ and } u') \]
The speed \( c \) can be calculated as follows:

\[ c = v' \cdot \left[ (\lambda \cos \phi)^2 + (\sin \phi)^2 \right]^{1/2} \]

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**Fig. 1** Different types of *Darrieus* wind turbines
Fig. 2 Shape and aero dynamical characteristics of the symmetrical profile NACA 0012

Fig. 3 Aerodynamic forces acting on an blade element of a *Darrieus* rotor
(NB: A is perpendicular and W is parallel to the vector c)
Remember that the angle of attack of the profile \( \alpha \) changes its value during rotation of the blades. At each point (angle \( \varphi \)), the tangential force \( K_T \) acting on the blades is the resultant of the tangential components of the vectors lift force (A) and drag force (W), i.e.:

\[
K_T = A \cdot \sin \alpha - W \cdot \cos \alpha = [c_a \cdot \sin \alpha - c_w \cdot \cos \alpha] \cdot \frac{1}{2} \cdot \rho \cdot f \cdot c^2 \quad [\text{N}]
\]

where

- \( c_a \) = lift coefficient of the chosen symmetric profile
- \( c_w \) = drag coefficient of the chosen symmetric profile (for example de one in Fig. 2)
- \( \rho \) = average air density (approx. 1,25 kg/m\(^3\))
- \( f \) = surface of the blade [m\(^2\)]

**NB:** To obtain \( K_T \) in Newtons [N], the velocity \( c \) has to be introduced in [m/s]

The angle of attack \( \alpha \) can be calculated as follows:

\[
\alpha = \arctan \left( \frac{\sin \varphi}{\lambda + \cos \varphi} \right)
\]

At the positions 1 and 3 (Fig. 4), the angle \( \alpha \) is zero, so here the blades have no lift force, i.e. they do not produce energy.

Note: When a Darrieus rotor is connected to the grid, the velocity \( u' \) is a constant, since the generator is running synchronously with the grids constant frequency (50 Hz in Europe, 60 Hz in USA).
Fig. 4
Behavior of the velocities $v'$, $u'$ and $c$ who attack the blades of a Darrieus rotor during a complete rotation around its vertical axis.

Fig. 5 shows the variation of the angle of incidence (attack) $\alpha$ of the profile depending on the angle of rotation $\varphi$ of the rotor, being positive for $0^\circ < \varphi < 180^\circ$ and negative for $180^\circ < \varphi < 360^\circ$. However, although the incidence angle is negative, the blade will continue to act on a positive tangential traction, i.e., for $180^\circ < \varphi < 360^\circ$ the rotor will continue to produce energy.

The figures 6 and 7 show the lift coefficient and the drag coefficient of a symmetrical NACA profile.

The angle of incidence (attack) can be calculated as follows:

$$\alpha = \arctan \left[ \frac{\sin \varphi}{\lambda + \cos \varphi} \right]$$
Fig. 5 Angle of incidence \( \alpha \) of the profile depending on the angle of rotation \( \varphi \) of the rotor [Source: "Flow Modelling in a Darrieus Turbine for Moderate Reynoldsnumber – C. Ploestenu, D. Tarziu et T. Maitre"]

Fig. 6 Lift coefficient \( c_a \) depending on the angle of incidence \( \alpha \) of a symmetrical profile NACA [Source: "Flow Modelling in a Darrieus Turbine for Moderate Reynoldsnumber – C. Ploestenu, D. Tarziu et T. Maitre"]
Fig. 7 Drag coefficient $c_w$ depending on the angle of incidence $\alpha$ of a symmetrical profile NACA [Source: "Flow Modelling in a Darrieus Turbine for Moderate Reynolds number – C. Ploestenu, D. Tarziu et T. Maitre"]

The power of a Darrieus wind turbine

If in equation [2], for the speed $c$ we introduce the value provided by equation [1], we get

$$K_T = \frac{1}{2} \cdot \rho \cdot f \cdot v^2 \cdot [c_a \cdot \sin \alpha - c_w \cdot \cos \alpha] \cdot [(\lambda + \cos \phi)^2 + (\sin \phi)^2]$$

($(f = \text{surface of the blade})$)

For a given wind speed $v$, the product $\frac{1}{2} \cdot \rho \cdot f \cdot v^2$ is a constant $k$, so that we can write

$$f(K_T) = k \cdot f(\lambda, c_a, c_w, \alpha, \phi)$$

With the help of the figures 5, 6 and 7, we can now calculate the function

$$f(\lambda, c_a, c_w, \alpha, \phi) = [c_a \cdot \sin \alpha - c_w \cdot \cos \alpha] \cdot [(\lambda + \cos \phi)^2 + (\sin \phi)^2]$$

We did so, and we have obtained the curve $f(K_T) = f(\phi)$ shown in Fig. 8.
Therefore, the torque, and with it the power, of the Darrieus rotor will oscillate in a similar way.

Fig. 9 shows the torque coefficient $c_m$ of a 3-blade Darrieus turbine.

With this coefficient one can calculate the torque of the Darrieus turbine with vertical blades (Fig. 1, left):

$$M = \frac{1}{4} \cdot c_m \cdot \rho \cdot H \cdot D^2 \cdot v^2 \quad [\text{Nm}]$$

and its power:

$$P = \left(\frac{2 \cdot \pi \cdot n}{60}\right) \cdot M \quad [\text{W}]$$

where

$v$ = wind speed [m/s]
$D$ = diameter of the Darrieus turbine (vertical blades) [m]
$H$ = height of the Darrieus turbine (= length of the blades) [m]
n = rotation speed of the Darrieus turbine [r.p.m.]

(NB: In a Darrieus wind turbine connected to the electrical grid, n is constant)

**Fig. 9** Torque coefficient $c_m$ of a Darrieus turbine with 3 blades [Source: Lain & Osorio: Simulation of a straight-bladed Darrieus-type cross flow turbine]
**Conclusion:**

The power of a wind turbine varies as shown in Fig. 9. For certain angles of rotation $\phi$ its power is negative. For a Darrieus wind turbine connected to the electrical grid, this means that at times the rotor absorbs energy from the grid, i.e., the synchronous generator connected to it via a gear as a motor.

Finally, saying that the study of the aerodynamics of a Darrieus turbine is quite fascinating.

**Appendix: Optimum width of the blades of a Darrieus rotor**

In case someone wants to build his own Darrieus wind turbine, we reproduce the formula given by [Le Gourières] to calculate the optimum width $t$ of the blades:

$$t = \frac{(5\cdot R)}{(z\cdot \lambda_d^2)}$$

where

$R$ is the radius of the rotor (in the rotors with parabolic shape, $R$ is the maximum radius)

$z$ is the number of blades (generally 2 or 3)

$\lambda_d$ is the rated tip-speed ratio of the rotor (between 5 and 7)

Example:

Cylindrically shaped Darrieus Rotor of 2.5 meters diameter with 3 blades. Rated tip-speed ratio $\lambda_d = 5$

$R = D/2 = 1.25 \text{ m}$

$\begin{align*}
t &= \frac{(5\cdot1.25)}{(3\cdot25)} = 0.083 \text{ m} = 8.3 \text{ cm}
\end{align*}$


In the Web page [www.amics21/laveritat.htm](http://www.amics21/laveritat.htm) you will also find, among many other things, some manuals for building a Savonius wind generator with the two halves of an old oil barrel of 200 liters or a horizontal axis wind turbine with a car alternator.